

# Configurational entropy as a bounding of Gauss-Bonnet braneworld models

R. A. C. Correa<sup>1\*</sup>, P. H. R. S. Moraes<sup>2†</sup>, A. de Souza Dutra<sup>1‡</sup>, W. de Paula<sup>2§</sup>, and T. Frederico<sup>2¶</sup>

<sup>1</sup>*UNESP, Universidade Estadual Paulista,  
12516-410, Guaratinguetá, SP, Brazil and  
<sup>2</sup>ITA, Instituto Tecnológico de Aeronáutica,  
12228-900, São José dos Campos, SP, Brazil*

## Abstract

Configurational entropy has been revealed as a reliable method for constraining some parameters of a given model [Phys. Rev. D **92** (2015) 126005, Eur. Phys. J. C **76** (2016) 100]. In this letter we calculate the configurational entropy in Gauss-Bonnet braneworld models. Our results restrict the range of acceptability of the Gauss-Bonnet scalar values. In this way, the information theoretical measure in Gauss-Bonnet scenarios opens a new window to probe situations where the additional parameters, responsible for the Gauss-Bonnet sector, are arbitrary. We also show that such an approach is very important in applications that include p and Dp-branes and various superstring-motivated theories.

Keywords: Entropy, Gauss-Bonnet, Gravity

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\* fis04132@gmail.com

† moraes.phrs@gmail.com

‡ dutra@feg.unesp.br

§ wayne@ita.br

¶ tobias@ita.br

## I. INTRODUCTION

In the late 90's of the last century, observational evidences of an accelerated expansion of the universe were found [1, 2]. It was proposed, as an explanation for such a counterintuitive phenomenon, the existence of the vacuum quantum energy, which would manifest in large scales as an “anti-gravitational force”. The vacuum quantum energy appears in the dynamical equations of the universe in the  $\Lambda$ CDM (or standard) cosmological model as the cosmological constant (CC) density parameter  $\Omega_\Lambda$ . However, when one compares the value of  $\Omega_\Lambda$  which can account for the present dynamical scenario of the universe [3] with the value theoretically predicted in Particle Physics [4], one realizes a huge discrepancy between them, which is known as the CC problem.

An alternative to evade the CC problem - among others surrounding the standard cosmology model, which we shall quote below - is to consider modified theories of gravity. The  $f(R)$  [5]-[7] and  $f(R, T)$  [8, 9] theories of gravity have generated possibilities of describing the cosmic acceleration with no need of a CC, evading, in this way, the CC problem. Those theories consider the gravitational part of their action to be dependent on a function of the Ricci scalar  $R$  or of both  $R$  and  $T$ , with  $T$  being the trace of the energy-momentum tensor.

Another possibility of describing cosmic acceleration without a CC comes from cosmology derived from Gauss-Bonnet (GB) gravity [10]-[13]. Those are obtained from the consideration of the GB invariant or a function of it in the gravitational part of the action. It is known that higher orders in the GB term naturally arise in the low energy limit of string theory [14].

Some alternative gravity models are a powerful tool to evade the hierarchy problem as well, which is related to the large discrepancy among aspects of the gravitational force and the other fundamental forces. Those are the braneworld models [15]-[18], which consider our observable universe as a  $3 + 1$  hypersurface (the brane) embedded in a five-dimensional space named *bulk*. Gravity, departing from the other forces, would be able to propagate through the bulk, justifying the hierarchy in the four-dimensional universe.

It is possible to unify some of the above formalisms in order to evade more than one standard cosmology shortcoming simultaneously by invoking the brane set up in  $f(R)$  [19]-[24],  $f(R, T)$  [25, 26] and GB [27]-[29] gravity models.

The alternative gravity models mentioned above carry some “free” parameters with them whose values can be constrained, for instance, by cosmological observations. Observational constraints in  $f(R)$  gravity parameters can be checked in [31]-[36]. For observational restrictions in  $f(R, T)$  and GB theories, check [37] and [38]-[41], respectively.

On the other hand, in a recent work [42], the concept of entropy has been reintroduced in the literature, by taking into account the dynamical and informational contents of models with localized energy configurations. Based on the Shannon's information entropy, the so-called Configurational Entropy (CE) was constructed. It can be applied to several nonlinear scalar field models featuring solutions with spatially-localized energy. As pointed out in [42], the CE can resolve situations where the energies of the configurations are degenerate. In this case, the CE can be used to select the best configuration. Furthermore, the authors pointed out that this information-entropic measure is an essential tool in the study of complex spatially-localized configurations.

We are going to discourse about the CE mechanism below. For now, it is interesting to highlight some of its applications which reveal CE as a powerful physical tool nowadays. For instance, it was shown in [43] that the CE quantifies the emergence of spatially-localized, time-dependent, long-lived structures known as oscillons [44–46]. In that case, the CE is responsible for providing the informational content of nonequilibrium field structures, in particular of coherent states that emerge during spontaneous symmetry breaking. By considering a Starobinsky functional form for  $f(R)$ , i.e.,  $f(R) = R + \alpha R^2$ , in brane models with nonconstant curvature, the  $\alpha$  parameter values were constrained by CE consideration in [47]. The free parameters of the  $f(R, T) = R - \alpha T$  and  $f(R, T) = R + \beta R^2 - \alpha T$  brane models were constrained in [48]. It has been shown that, indeed, CE can be used in order to extract a rich information about the structure of the model configurations. Moreover, CE was applied to pure brane models in [49] and restrictions to the anti-de-Sitter bulk curvature and domain wall thickness were obtained.

Studies regarding CE can also be found in solitonic  $Q$ -balls [50], in the context of  $(2 + 1)$ -dimensional Ginzburg-Landau models [51], in astrophysical objects [52], in two interacting scalar fields theories [53], in traveling solitons in Lorentz and CPT breaking systems [54], and in topological Abelian string-vortex and string-cigar context [55].

Our intention in this letter is to obtain some restrictions to the GB braneworld parameters via CE approach. The letter is organized as follows. In Section II we discourse about the information content which can be obtained from the CE approach. Since we are interested in restricting GB braneworld models, we present a brief review of those in Section III. In Section IV we calculate the CE in GB braneworld and we discuss our results in Section V.

## II. THE CONFIGURATIONAL ENTROPY

Gleiser and Stamatopoulos (GS) [42] have recently proposed a detailed picture of the so-called CE for the structure of localized solutions in classical field theories. In this section, analogously to that work, we formulate a CE measure in the functional space, from the field configurations where the GB braneworld scenarios can be studied.

There is an intimate link between information and dynamics, where the entropic measure plays a prominent role. The entropic measure is well known to quantify the informational content of physical solutions to the equations of motion and their approximations, namely, the CE in functional space [42]. GS proposed that nature optimizes not solely by optimizing energy through the plethora of *a priori* available paths, but also from an informational perspective [42].

The starting point is to consider structures with spatially-localized energy and a modal fraction  $f(\omega)$  which measures the relative weight of each mode  $\omega$  such that

$$f(\omega) = \frac{|\mathcal{F}[\omega]|^2}{\int d\omega |\mathcal{F}[\omega]|^2}, \quad (1)$$

with  $\mathcal{F}(\omega)$  being the Fourier transform.

The CE is defined as

$$\mathcal{S}_C(f) = - \sum f_n \ln f_n \quad (2)$$

and provides the informational content of configurations compatible with the particular constraints of a given physical system. We can say that when all  $N$  modes carry the same weight,  $f_n = 1/N$  and the discrete CE presents a maximum at  $\mathcal{S}_C = \ln N$ . Alternatively, if only one mode is present,  $\mathcal{S}_C = 0$ .

For general, non-periodic functions in an open interval, the continuous CE reads

$$\mathcal{S}_C(f) = - \int d\omega \hat{f}(\omega) \ln |\hat{f}(\omega)|, \quad (3)$$

with  $\hat{f}(\omega) \equiv f(\omega)/f_{max}(\omega)$  defined as the normalized modal fraction, whereas  $f_{max}(\omega)$  is the maximum fraction. In this case, this condition ensures the positivity of  $\mathcal{S}_C$ .

### III. THE GAUSS-BONNET BRANEWORLD GRAVITY MODEL

Since we are interesting in the calculation of the CE in GB brane models, it is worth to briefly review such an alternative gravity theory. This can be appreciated in the following.

An alternative to extend standard gravity is through the addition of the GB term

$$G = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}, \quad (4)$$

or a function of it, in the usual Einstein-Hilbert gravity lagrangian. In (4),  $R_{\mu\nu}$  is the Ricci tensor and  $R_{\mu\nu\lambda\rho}$  is the Riemann tensor. Since we are considering a five-dimensional braneworld model, the Greek indices above assume the values 0, 1, 2, 3, 4.

The GB brane gravity action for a general function of the GB term reads:

$$S = \frac{1}{2} \int d^4x dy \sqrt{-g} [R + h(G)], \quad (5)$$

with  $g$  being the determinant of the metric and  $h(G)$  being a function of the GB scalar.

We consider as the matter source of the universe a scalar field  $\phi$  specified by the lagrangian density

$$\mathcal{L} = \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \quad (6)$$

to be added to (5), with  $V(\phi)$  being the scalar field potential.

Moreover we will work with the following metric

$$ds^2 = e^{2A(y)} \eta_{ab} dx^a dx^b - dy^2, \quad (7)$$

with  $A(y)$  representing the warp function,  $\eta_{ab}$  the Minkowski metric and  $a, b$  running from 0 to 3.

In [28], the authors showed that for the metric (7),

$$G = 24(4A''A'^2 + 5A'^4) \quad (8)$$

and the energy-momentum tensor

$$T_{ab} = \eta_{ab} \left[ \frac{1}{2} \phi'^2 + V(\phi) \right] e^{2A}, \quad (9)$$

with primes denoting derivatives with respect to  $y$ .

Now, after some straightforward calculation it is possible to obtain the following equations of motion [28]:

$$-3A'' = 2\phi'^2 + 12h_G(G)A''A'^2, \quad (10)$$

$$6A'^2 = \phi'^2 - 2V(\phi) - \frac{1}{2}h(G) + 48h_G(G)(A'^4 + A''A'^2), \quad (11)$$

with  $h_G(G) \equiv dh(G)/dG$ . Note that within this methodology, the potential  $V(\phi)$  is a variable to be determined instead of a quantity introduced as a basic characteristic of the theory.

For the case  $h(G) = \alpha G^n$ , with  $\alpha$  and  $n$  being constants, it can be shown that the explicit form assumed by the potential is

$$\begin{aligned} V(\phi) = & -\frac{3}{4}A' + 3A'^2 - \frac{1}{4}\alpha G^n + 3\alpha n A'^2(8A'^2 + 7A'')G^{n-1} \\ & -\alpha n(n-1)[A'(14A'^2 + 2A'')G' + (2A'^2 - A'')G'']G^{n-2} \\ & -\alpha n(n-1)(n-2)G'^2(2A'^2 - A'')G^{n-3}. \end{aligned} \quad (12)$$

Furthermore, we have

$$\begin{aligned} \phi'^2 = & -\frac{3}{2}A'' - 6\alpha n A'^2 A'' G^{n-1} \\ & -2\alpha n(n-1)[(2A'^2 - A'')G'' - 2A'(A'^2 - 5A'')G']G^{n-2} \\ & -2\alpha n(n-1)(n-2)G'^2(2A'^2 - A'')G^{n-3}. \end{aligned} \quad (13)$$

On the other hand, from (9) the energy density  $T_{00}$  is

$$\rho = e^{2A} \left[ \frac{1}{2} \phi'^2 + V(\phi) \right]. \quad (14)$$

Following references [28, 56], we can choose the ansatz

$$A(y) = B \ln [\operatorname{sech}(y)], \quad (15)$$

where  $B > 0$ .

Now, using the Eqs.(8), (12), (13) and (15), the energy density (14) is written in the form

$$\rho(y) = \sum_{\ell=1}^5 s_{\ell}(n, B, \alpha) Q_{\ell}(n, B, \alpha; y), \quad (16)$$

where we are using the following definitions

$$s_1(n, B, \alpha) \equiv \frac{3B}{2}, \quad Q_1(n, B, \alpha; y) \equiv \text{sech}^{2B}(y), \quad (17)$$

$$s_2(n, B, \alpha) \equiv -3B^2, \quad Q_2(n, B, \alpha; y) \equiv \text{sech}^{2B}(y) \tanh^2(y), \quad (18)$$

$$s_3(n, B, \alpha) \equiv \alpha n 2^{3n} 3^n B^{3n+1}, \quad Q_3(n, B, \alpha; y) \equiv \text{sech}^{2B}(y) \tanh^{2n+2}(y) [\Psi_B(y)]^{n-1}, \quad (19)$$

$$s_4(n, B, \alpha) \equiv -\alpha n 2^{3n-2} 3^{n+1} B^{3n-1}, \quad Q_4(n, B, \alpha; y) \equiv \text{sech}^{2B+2}(y) \tanh^{2n}(y) [\Psi_B(y)]^{n-1}, \quad (20)$$

$$s_5(n, B, \alpha) \equiv -\alpha 2^{3n-2} 3^n B^{3n}, \quad Q_5(n, B, \alpha; y) \equiv \text{sech}^{2B}(y) \tanh^{2n}(y) [\Psi_B(y)]^n, \quad (21)$$

with

$$\Psi_B(y) \equiv 5B \tanh^2(y) - 4 \text{sech}^2(y). \quad (22)$$

The profiles for the energy density and warp factor are depicted in Fig.1, which shows the influence of the  $B$  parameter on the configurations. It is important to explain the reason for working with small values of  $B$ . In Fig.1, the reader can note that when  $B$  increases the brane is narrowed down. Moreover, simultaneously, the energy density develops lateral peaks and a central valley which goes rapidly to zero as  $B$  increases. Such a critical phenomenon of thick brane models is called “brane splitting” and it was first presented in [57]. It has already appeared in GB [20] and  $f(R)$  [21] models. Since in the brane splitting case, the field will not be confined to the brane, we avoid large values of  $B$ .

#### IV. INFORMATION CONTENT IN GAUSS-BONNET BRANEWORLD

Now we will apply the new concept of CE in the functional space from the field configurations where the GB braneworld scenarios can be studied. To begin, we write the Fourier transform

$$\mathcal{F}[\omega] = \frac{1}{\sqrt{2\pi}} \int dy e^{i\omega y} \rho(y), \quad (23)$$

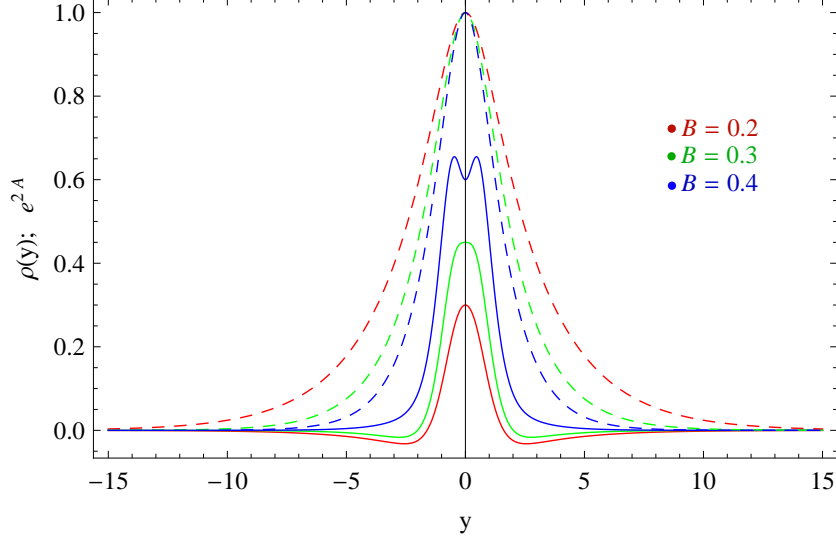


FIG. 1: Energy density (thin continuous line) and warp factor (dashed line) with  $\alpha = 1$  and  $n = 1$ .

where  $\rho(y)$  is the standard energy density. Here, it is important to remark that the energy density given by Eq.(16) is localized. In fact, as shown by GS, the condition necessary so that the CE is well-defined in physical applications is that the energy density is represented by a spatially-confined structure.

By using the Plancherel theorem, it follows that

$$\int d\omega |\mathcal{F}[\omega]|^2 = \int dr |\rho(y)|^2. \quad (24)$$

Now, substituting the energy density given by Eq.(16) into Eq.(23), we can obtain, after some arduous calculation, the following Fourier transform

$$\mathcal{F}(\omega) = \sum_{\ell=1}^5 \sum_{j=1}^2 \tilde{s}_{\ell}(n, B, \alpha) \mathcal{I}_{\ell,j}(n, B, \alpha; \omega), \quad (25)$$

where  $\tilde{s}_{\ell}(n, B, \alpha) \equiv s_{\ell}(n, B, \alpha) / \sqrt{2\pi}$  and  $\mathcal{I}_{\ell,j}[n, B, \alpha; \omega]$  are the following corresponding functions

$$\mathcal{I}_{1,1}(n, B, \alpha; \omega) = \frac{4}{c + i\omega} \mathcal{G} \left[ c, \frac{c + i\omega}{2}; \frac{c + i\omega}{2} + 1; -1 \right], \quad (26)$$

$$\mathcal{I}_{1,2}(n, B, \alpha; \omega) = \frac{4}{c - i\omega} \mathcal{G} \left[ c, \frac{c - i\omega}{2}; \frac{c - i\omega}{2} + 1; -1 \right], \quad (27)$$



$$\begin{aligned}\mathcal{I}_{2,1}(n, B, \alpha; \omega) &= \frac{1}{2 + \tilde{b}_0 + i\omega} \mathcal{G} \left[ \tilde{b}_0, \frac{2 + \tilde{b}_0 + i\omega}{2}; \frac{4 + \tilde{b}_0 + i\omega}{2}; -1 \right] \\ &+ \frac{1}{b_0 - i\omega} \mathcal{G} \left[ \tilde{b}_0, \frac{b_0 - i\omega}{2}; \frac{b_0 - i\omega}{2} + 1; -1 \right],\end{aligned}\quad (28)$$

$$\begin{aligned}\mathcal{I}_{2,2}(n, B, \alpha; \omega) &= \frac{1}{b_0 + i\omega} \mathcal{G} \left[ \tilde{b}_0, \frac{b_0 + i\omega}{2}; \frac{b_0 + i\omega}{2} + 1; -1 \right] \\ &+ \frac{1}{i\omega - 2 - \tilde{b}_0} \mathcal{G} \left[ \tilde{b}_0, \frac{i\omega - 2 - \tilde{b}_0}{2}; \frac{i\omega - \tilde{b}_0}{2}; -1 \right],\end{aligned}\quad (29)$$

$$\mathcal{I}_{3,1}(n, B, \alpha; \omega) = \frac{(\alpha_1 \alpha_2)^{n-1} \Gamma[\gamma_1 + 1] \Gamma[\gamma_2 + 1]}{2\Gamma[\gamma_1 + \gamma_2 + 2]} \mathcal{H}_D^{(3)}[\gamma_1 + 1; \gamma_3, -\gamma_4, -\gamma_4; \gamma_1 + \gamma_2 + 2; -1, v, u], \quad (30)$$

$$\mathcal{I}_{3,2}(n, B, \alpha; \omega) = \frac{(\alpha_1 \alpha_2)^{n-1} \Gamma[1 - \bar{\gamma}_0] \Gamma[\bar{\gamma}_1 + 1]}{2\Gamma[\bar{\gamma}_1 - \bar{\gamma}_0 + 2]} \mathcal{H}_D^{(3)}[1 - \bar{\gamma}_0; \bar{\gamma}_2, -\bar{\gamma}_3, -\bar{\gamma}_4; \bar{\gamma}_1 - \bar{\gamma}_0 + 2; -1, v, u], \quad (31)$$

$$\mathcal{I}_{4,1}(n, B, \alpha; \omega) = \frac{2^{\delta-1} \xi_1^\theta (\epsilon_1 \epsilon_2)^\theta \Gamma[R] \Gamma[2\zeta + R + 1]}{\Gamma[2\zeta + R + 1]} \mathcal{H}_D^{(3)}[R; \Phi, -\theta, -\theta; 2\zeta + R + 1; -1, \tilde{v}, \tilde{u}], \quad (32)$$

$$\mathcal{I}_{4,2}(n, B, \alpha; \omega) = \frac{2^{\tilde{\delta}-1} \tilde{\xi}_1^{\tilde{\theta}} (\epsilon_1 \epsilon_2)^{\tilde{\theta}} \Gamma[\tilde{R}] \Gamma[\tilde{\lambda} + 1]}{\Gamma[\lambda + \tilde{R} + 1]} \mathcal{H}_D^{(3)}[\tilde{R}; \Phi, -\theta, -\theta; \lambda + R + 1; -1, \hat{v}, \hat{u}], \quad (33)$$

$$\mathcal{I}_{5,1}(n, B, \alpha; \omega) = \frac{2^{\hat{\delta}-1} \hat{\xi}_1^{\hat{\theta}} (\tilde{\epsilon}_1 \tilde{\epsilon}_2)^\theta \Gamma[\hat{R}] \Gamma[2\hat{\zeta} + \hat{R} + 1]}{\Gamma[2\zeta + R + 1]} \mathcal{H}_D^{(3)}[\hat{R}; \hat{\Phi}, -\hat{\theta}, -\hat{\theta}; 2\hat{\zeta} + \hat{R} + 1; -1, U, Y], \quad (34)$$

$$\mathcal{I}_{5,2}(n, B, \alpha; \omega) = \frac{2^{\mathring{\delta}-1} \mathring{\xi}_1^{\mathring{\theta}} (\tilde{\epsilon}_1 \tilde{\epsilon}_2)^{\mathring{\theta}} \Gamma[\mathring{R}] \Gamma[\mathring{\lambda} + 1]}{\Gamma[\lambda + \mathring{R} + 1]} \mathcal{H}_D^{(3)}[\mathring{R}; \mathring{\Phi}, -\mathring{\theta}, -\mathring{\theta}; \lambda + \mathring{R} + 1; -1, U, Y], \quad (35)$$

with the definitions

$$\begin{aligned}
c &\equiv 2B + 2, \quad b_0 \equiv 2B, \quad \tilde{b}_0 \equiv 2B + 2, \quad \alpha_1 \equiv \frac{d_0}{2} + \sqrt{\frac{d_0^2}{4} - 1}, \quad \alpha_2 \equiv \frac{d_0}{2} - \sqrt{\frac{d_0^2}{4} - 1}, \\
d_0 &\equiv \frac{5B}{2} + 4, \quad \epsilon_1 \equiv \frac{D_0}{2} + \sqrt{\frac{D_0^2}{4} - 1}, \quad \epsilon_2 \equiv \frac{D_0}{2} - \sqrt{\frac{D_0^2}{4} - 1}, \quad D_0 \equiv 2 + \frac{4}{5B}, \\
\gamma_1 &\equiv \frac{2B - 4n + 4 + i\omega}{2} - 1, \quad \gamma_2 \equiv 2n + 2, \quad \gamma_3 \equiv 2B + 4, \quad \gamma_4 \equiv n - 1, \quad v \equiv \frac{1}{\alpha_1}, \quad u \equiv \frac{1}{\alpha_2}, \\
\bar{\gamma}_0 &\equiv \frac{2B - 4n + 4 - i\omega}{2} - 1, \quad \bar{\gamma}_1 \equiv 2n + 2, \quad \bar{\gamma}_2 \equiv -2B - 4, \quad \bar{\gamma}_3 = \bar{\gamma}_4 \equiv n - 1, \quad \xi_1 \equiv 5B, \\
\zeta &\equiv n, \quad \theta \equiv n - 1, \quad \Phi \equiv 2\theta + 2B + 2 + 2n, \quad R \equiv i\omega - 2\zeta - 2\theta + \Phi, \quad U \equiv \frac{1}{\tilde{\epsilon}_2}, \quad Y \equiv \frac{1}{\tilde{\epsilon}_1}, \\
\delta &\equiv 2B + 2, \quad \tilde{v} = \hat{v} \equiv \frac{1}{\epsilon_1}, \quad \tilde{u} = \hat{u} \equiv \frac{1}{\epsilon_2}, \quad \tilde{R} = -\hat{R} \equiv i\omega - 2\zeta + 2\theta + \Phi, \\
\hat{\Phi} = -\check{\Phi} &\equiv 2\theta - 2B + 2 + 2n, \quad \hat{\theta} = -\check{\theta} \equiv 2n - 1, \quad \check{R} \equiv -i\omega - 2\zeta + 2\theta + \Phi.
\end{aligned}$$

Furthermore, in the above expressions,  $\mathcal{G}[\odot, \odot; \odot; \odot]$  stand for the well-known hypergeometric functions and  $\mathcal{H}_D^{(3)}[\otimes; \otimes, \otimes; \otimes; \otimes, \otimes, \otimes]$  is the so-called Lauricella functions of three variables [58].

Thus, the modal fraction (1) becomes

$$f(\omega) = \frac{\sum_{\ell, \ell'=1}^5 \sum_{j, j'=1}^2 \tilde{s}_\ell(n, B, \alpha) \tilde{s}_{\ell'}^*(n, B, \alpha) \mathcal{I}_{\ell, j}(n, B, \alpha; \omega) \mathcal{I}_{\ell', j'}^*(n, B, \alpha; \omega)}{\sum_{\ell, \ell'=1}^5 \sum_{j, j'=1}^2 \tilde{s}_\ell(n, B, \alpha) \tilde{s}_{\ell'}^*(n, B, \alpha) \int d\omega \mathcal{I}_{\ell, j}(n, B, \alpha; \omega) \mathcal{I}_{\ell', j'}^*(n, B, \alpha; \omega)}. \quad (36)$$

In Fig.2 the modal fraction is depicted for different values of the parameter  $B$ .

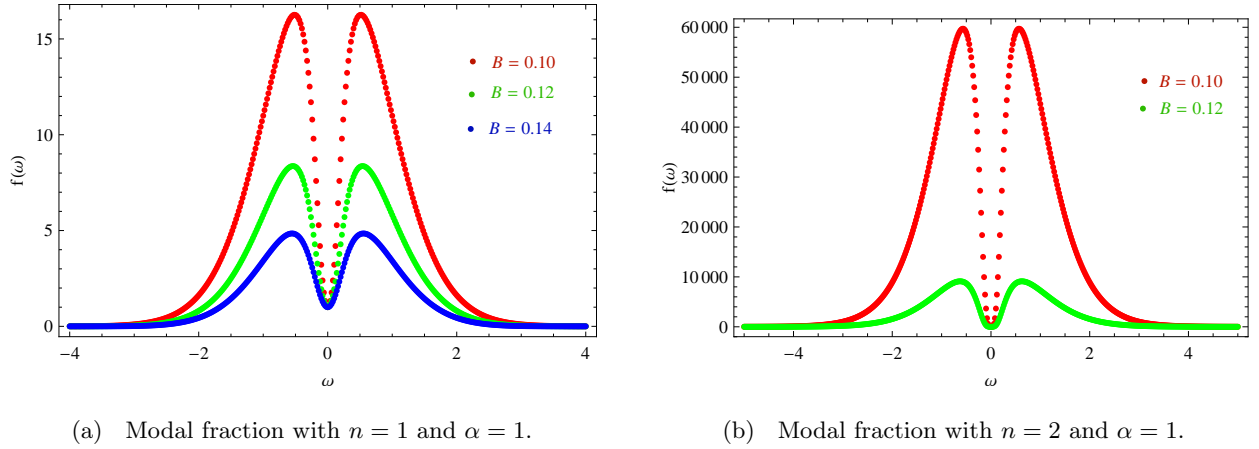


FIG. 2: Modal fractions

In Fig.3 it can be seen that given a value of  $B$  there is a value of  $\alpha$  for which CE is a minimum. Furthermore when  $B$  decreases, that minimum value also decreases. This results are in agreement with the CE concept found in [42]. Here, we can observe that lower CE correlates with lower energy. Thus, the most prominent solutions to the GB theory under analysis are given for some specific values of  $\alpha$ . This will be analysed further in the next section.

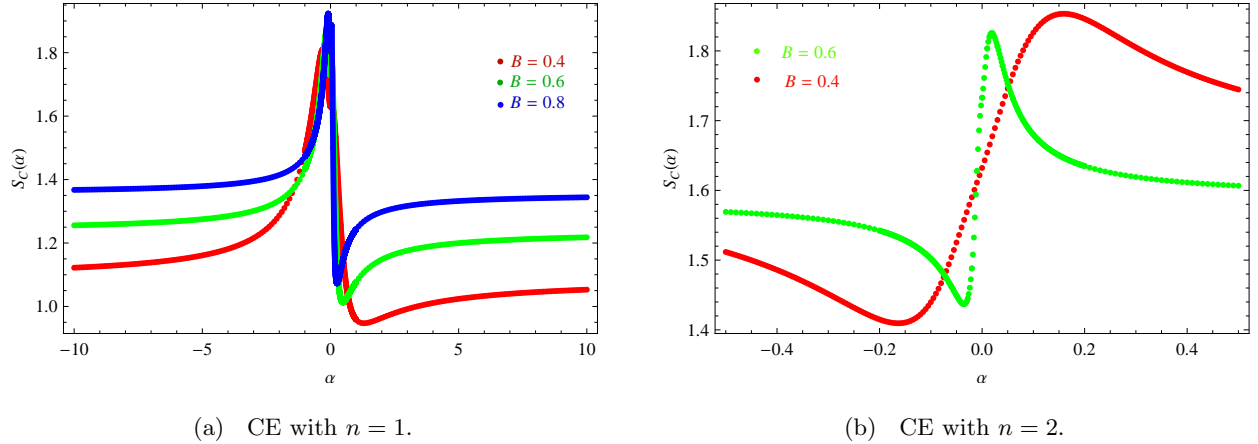


FIG. 3: Configurational entropy

## V. DISCUSSION AND CONCLUSIONS

The CE has been revealed as a reliable method for constraining some given model parameters, as one can check, for instance, in [47, 48]. The entropic information has been here studied in braneworld models, with emphasis on the GB scenario, which has been chosen by its very physical content and usefulness. We have shown that the information theoretical measure of GB braneworld models opens new possibilities to physically constrain, for example, parameters that are related to the GB term. The CE provides the most appropriate value of these parameters that are consistent with the best organizational structure.

The information measure of the system organization is related to modes in the braneworld model. Hence the constraints of the parameters that we obtained for the GB model provide the range of the parameters associated to the most organized braneworld models with respect to the information content of these models.

By analysing Fig.3 of the previous section, we can see that for high values of  $B$ , the CE minimum, which is related to the reliable solutions of the system, is found for  $\alpha = 0$  or  $\alpha \simeq 0$ . In other words, there is a specific value for  $B$  which makes the GB term to vanish. Therefore, the CE serves as a new tool to specify the dynamic constraints of the GB model.

As we can see, the CE provides a complementary perspective to investigate alternative theories of gravity. Further interests that concerns to CE and which we are interested in, can be found in dynamical bound in alternatives theory of gravity, such as Brans-Dicke [59], Kaluza-Klein [60] and  $f(R, L_m)$  [61] gravity models, among many others.

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